Nondeterministic Discontinuous Lambek Calculus

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Graded prosodic algebra

basic prosodic algebra:

- (L,+, 0)
- L is a set, 0 ∈ L
- + is a binary operation on L

$$\begin{aligned} s_1+(s_2+s_3) &= (s_1+s_2)+s_3 && \text{associativity} \\ 0+s = \quad s &= s+0 && \text{identity} \end{aligned}$$

graded prosodic algebra:

- (L,+, 0, 1)
- 1 is a separator

Discontinuous prosodic structure

$$(L_0, L_1, L_2, \ldots, +, 0, 1; \times)$$

$$\begin{array}{lll} + & : & L_i \times L_j \longrightarrow L_{i+j} \text{ as in the graded prosodic algebra} \\ \times & : & L_{i+1} \times L_j \times L_{i+j} \text{ is the smallest relation such that} \\ & & \forall s_1{+}1{+}s_3 \in L_{i+1}, s_2 \in L_j, \times(s_1{+}1{+}s_3, s_2, s_1{+}s_2{+}s_3) \end{array}$$

New operators

$\hat{}$ 'bridge'

$\check{}$ 'split'

$\downarrow$ 'infix'

$\uparrow$ 'extract'

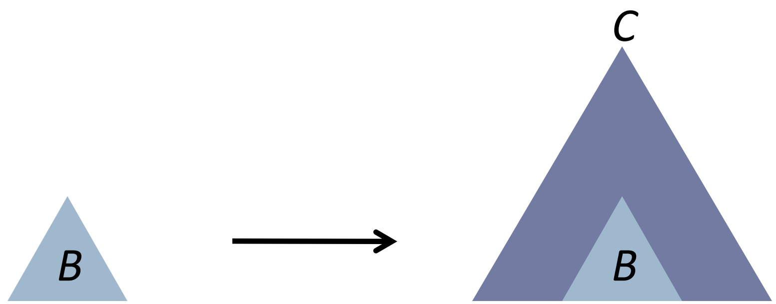
$\odot$ 'discontinuous product'

Example of operators

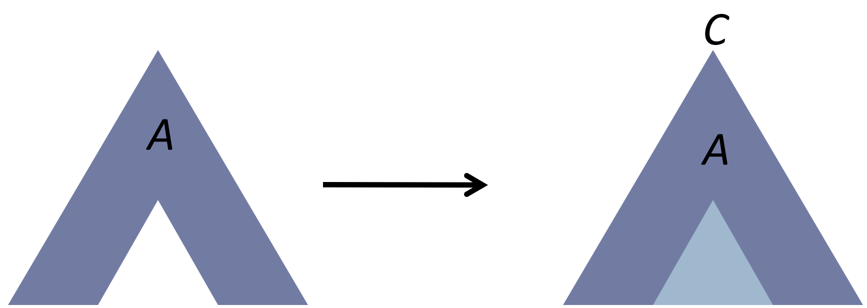
$$If\ A = a + b\ \ then\ \check{A} = a + 1 + b$$

$$If\ A = a + 1 + b\ \ then\ \hat{A} = a + b$$

$C \uparrow B ::$



$A \downarrow C ::$



Interpretation of operators

$$
\begin{array}{rcl}
[[\check{}\,B]] & = & \{s \mid \forall s', \times(s,0,s') \Rightarrow s' \in [[B]]\} \\
[[\hat{}\,A]] & = & \{s' \mid \exists s \in [[A]], \times(s,0,s')\} \\
[[A{\downarrow}C]] & = & \{s_2 \mid \forall s_1 \in [[A]], \times(s_1,s_2,s) \Rightarrow s \in [[C]]\} \\
[[C{\uparrow}B]] & = & \{s_1 \mid \forall s_2 \in [[B]], \times(s_1,s_2,s) \Rightarrow s \in [[C]]\} \\
[[A \odot B]] & = & \{s \mid \exists s_1 \in [[A]], s_2 \in [[B]], \times(s_1,s_2,s)\}
\end{array}
$$

$$B \subseteq A{\downarrow}C \quad \text{iff} \quad A \odot B \subseteq C \quad \text{iff} \quad A \subseteq C{\uparrow}B$$

$$A \subseteq \check{}\,B \quad \text{iff} \quad \hat{}\,A \subseteq B$$

Notations

$$\sqrt[0]{A} \bullet \{1\} \cdots \{1\} \bullet \sqrt[i]{A} = A$$

$\Delta|_i\Gamma$ is the result of replacing the i-th separator in Δ by Γ

N = noun phrase
CN = N without Det

Hypersequent Calculus

$$\frac{\Delta(\overrightarrow{B}) \Rightarrow \overrightarrow{C}}{\Delta(\overrightarrow{\vee B}|_i\Lambda) \Rightarrow \overrightarrow{C}} \vee L$$

$$\frac{\Delta|_1\Lambda \Rightarrow \overrightarrow{B} \qquad \cdots \qquad \Delta|_{S(B)}\Lambda \Rightarrow \overrightarrow{B}}{\Delta \Rightarrow \overrightarrow{\vee B}} \vee R$$

Hypersequent Calculus

$$\frac{\Delta(\overrightarrow{A}|_1\Lambda) \Rightarrow \overrightarrow{C} \qquad \ldots \qquad \Delta(\overrightarrow{A}|_{S(A)}\Lambda) \Rightarrow \overrightarrow{C}}{\Delta(\overrightarrow{\hat{}A}) \Rightarrow \overrightarrow{C}} \,\hat{}L$$

$$\frac{\Delta \Rightarrow \overrightarrow{A}}{\Delta|_i\Lambda \Rightarrow \overrightarrow{\hat{}A}} \,\hat{}R$$

Hypersequent Calculus

$$\frac{\Gamma \Rightarrow \overrightarrow{A} \qquad \Delta(\overrightarrow{C}) \Rightarrow \overrightarrow{D}}{\Delta(\Gamma|_i \overrightarrow{A{\downarrow}C}) \Rightarrow \overrightarrow{D}} \downarrow L$$

$$\frac{\overrightarrow{A}|_1\Gamma \Rightarrow \overrightarrow{C} \qquad \ldots \qquad \overrightarrow{A}|_{S(A)}\Gamma \Rightarrow \overrightarrow{C}}{\Gamma \Rightarrow \overrightarrow{A{\downarrow}C}} \downarrow R$$

Hypersequent Calculus

$$\frac{\Gamma \Rightarrow \overrightarrow{B} \qquad \Delta(\overrightarrow{C}) \Rightarrow \overrightarrow{D}}{\Delta(\overrightarrow{C{\uparrow}B}|_i\Gamma) \Rightarrow \overrightarrow{D}} \uparrow L$$

$$\frac{\Gamma|_1 \overrightarrow{B} \Rightarrow \overrightarrow{C} \qquad \ldots \qquad \Gamma|_{S(C)} \overrightarrow{B} \Rightarrow \overrightarrow{C}}{\Gamma \Rightarrow \overrightarrow{C{\uparrow}B}} \uparrow R$$

Natural Deduction

- Uitleg over notatie

$$\beta - \psi : \ B$$

- β = reeks woorden en seperators
- Ψ = λ-calculus term
- B = type

Natural Deduction

$$\begin{array}{c}\vdots\\ \dfrac{\beta - \psi : \check{}\, B}{\beta \,|_{i}\, 0 - \psi : B}\; E^{\check{}}\end{array}$$

Natural Deduction

$$\dfrac{\begin{array}{c}\vdots\\ \beta|_{1}0-\psi:B\end{array} \qquad \cdots \qquad \begin{array}{c}\vdots\\ \beta|_{S(B)}0-\psi:B\end{array}}{\beta-\psi:\check{}\,B}\; I\check{}$$

Natural Deduction

$$\dfrac{\begin{array}{c}\vdots \\ \gamma - \chi : \hat{\ } A\end{array} \quad \begin{array}{c}\overrightarrow{a} - x : A^i \\ \vdots \\ \delta(\overrightarrow{a}|_1 0) - \omega(x) : D\end{array} \quad \cdots \quad \begin{array}{c}\overrightarrow{a} - x : A^i \\ \vdots \\ \delta(\overrightarrow{a}|_{S(A)} 0) - \omega(x) : D\end{array}}{\delta(\gamma) - \omega(\chi) : D} \; E\hat{\ }^i$$

Natural Deduction

$$\begin{array}{c}\vdots\\ \alpha - \phi : A \\ \hline \alpha\,|_i\, 0 - \phi : \hat{}\, A\end{array}\; I\,\hat{}$$

Natural Deduction

$$\dfrac{\begin{array}{cc} \vdots & \vdots \\ \alpha - \phi : A & \gamma - \chi : A{\downarrow}C \end{array}}{\alpha|_i\gamma - (\chi\ \phi) : C}\ E{\downarrow}$$

Natural Deduction

$$\cfrac{\begin{matrix}\overrightarrow{a}-x:A^{i} \\ \vdots \\ \overrightarrow{a}|_{1}\gamma-\chi:C\end{matrix} \qquad \cdots \qquad \begin{matrix}\overrightarrow{a}-x:A^{i} \\ \vdots \\ \overrightarrow{a}|_{S(A)}\gamma-\chi:C\end{matrix}}{\gamma-\lambda x\chi:A{\downarrow}C}\; I{\downarrow}^{i}$$

Natural Deduction

$$\dfrac{\begin{array}{cc} \vdots & \vdots \\ \gamma - \chi : C \uparrow B & \beta - \psi : B \end{array}}{\gamma |_i \beta - (\chi\ \beta) : C}\ E\uparrow$$

Natural Deduction

$$\dfrac{\begin{matrix} \overrightarrow{b} - y{:}B^i & & \overrightarrow{b} - y{:}B^i \\ \vdots & & \vdots \\ \gamma|_1 \overrightarrow{b} - \chi{:}C & \cdots & \gamma|_{S(C)} \overrightarrow{b} - \chi{:}C \end{matrix}}{\gamma - \lambda x \chi{:}C{\uparrow}B} \; I{\uparrow}^i$$

Applications of DLC

- Medial extraction
- Discontinuous idioms
- Parentheticals
- Gapping
- Quantification
- Complement alternation
- Particle shift
- Cross-serial dependencies

Applications of DLC: Medial extraction

dog that Mary saw today

$$\textbf{that} \quad - \quad \lambda x \lambda y \lambda z[(x\ z) \wedge (y\ z)]$$

$$:= \quad (\mathrm{CN}\backslash\mathrm{CN})/\hat{}(\mathrm{S}\uparrow\mathrm{N})$$

Applications of DLC: Medial extraction

saw

$$\textbf{saw}-\mathit{see}{:}(N\backslash S)/N \qquad a-x{:}N \;\; i$$

$$E/$$

today

$$\textbf{saw}+a-(\mathit{see}\ x){:}N\backslash S \qquad \textbf{today}-\mathit{today}{:}(N\backslash S)\backslash(N\backslash S)$$

Mary

$$\textbf{Mary}-m{:}N \qquad \textbf{saw}+a+\textbf{today}-(\mathit{today}\ (\mathit{see}\ x)){:}N\backslash S$$

$$E\backslash$$

$$\textbf{Mary}+\textbf{saw}+a+\textbf{today}-(\mathit{today}\ (\mathit{see}\ x)\ m){:}S$$

$$I{\uparrow}^{i}$$

$$\textbf{Mary}+\textbf{saw}+1+\textbf{today}-\lambda x(\mathit{today}\ (\mathit{see}\ x)\ m){:}S{\uparrow}N$$

$$I\hat{}$$

$$N) \qquad \textbf{Mary}+\textbf{saw}+\textbf{today}-\lambda x(\mathit{today}\ (\mathit{see}\ x)\ m){:}\hat{}(S{\uparrow}N)$$

$$E/$$

$$\textbf{ay}-\lambda y\lambda z[(\mathit{today}\ (\mathit{see}\ z)\ m)\wedge(y\ z)]{:}CN\backslash CN$$

$$E\backslash$$

$$y\ (\mathit{see}\ z)\ m)\wedge(\mathit{dog}\ z)]{:}CN$$

Applications of DLC: Discontinuous idioms

$$\mathbf{gave}+1+\mathbf{the}+\mathbf{cold}+\mathbf{shoulder} \quad - \quad \mathit{shun} \\ := \quad (\mathrm{N}\backslash\mathrm{S})\uparrow\mathrm{N}$$

$$\dfrac{\dfrac{\text{gave}\ldots\text{the cold shoulder}}{\mathbf{gave}+1+\mathbf{the}+\mathbf{cold}+\mathbf{shoulder}-\mathit{shun}:(\mathrm{N}\backslash\mathrm{S})\uparrow\mathrm{N}} \quad \dfrac{\text{John}}{\mathbf{John}-j:\mathrm{N}}}{\mathbf{gave}+\mathbf{John}+\mathbf{the}+\mathbf{cold}+\mathbf{shoulder}-(\mathit{shun}\ j):\mathrm{N}\backslash\mathrm{S}}\ E\uparrow$$

Applications of DLC: Parentheticals

a. Fortunately, John has perseverance.

b. John, fortunately, has perseverance.

c. John has, fortunately, perseverance.

d. John has perseverance, fortunately.

Applications of DLC: Parentheticals

$$\dfrac{\dfrac{\dfrac{\text{John}}{\textbf{John}-j{:}\mathrm{N}} \quad \dfrac{\dfrac{\text{has}}{\textbf{has}-\mathit{have}{:}(\mathrm{N}\backslash \mathrm{S})/\mathrm{N}} \quad \dfrac{\text{perseverence}}{\textbf{perseverence}-\mathit{perseverence}{:}\mathrm{N}}}{\textbf{has+perseverence}-(\mathit{have\ perseverence}){:}\mathrm{N}\backslash \mathrm{S}}\,E/}{\dfrac{\textbf{John+has+perseverence}-(\mathit{have\ perseverence\ j}){:}\mathrm{S}}{\textbf{John+has+1+perseverence}-(\mathit{have\ perseverence\ j}){:}{}^{\vee}\mathrm{S}}\,I^{\vee}}\,E\backslash \quad \dfrac{\text{fortunately}}{\textbf{fortunately}-\mathit{fortunately}{:}{}^{\vee}\mathrm{S}{\downarrow}\mathrm{S}}}{\textbf{John+has+fortunately+perseverence}-(\mathit{fortunately}\ (\mathit{have\ perseverence\ j})){:}\mathrm{S}}\,E{\downarrow}$$

Applications of DLC: Gapping

John studies logic, and Charles, phonetics.

$$
\begin{array}{lcl}
\textbf{and} & - & \lambda x \lambda y \lambda z[(y\ z) \wedge (x\ z)] \\
 & := & ((\mathrm{S}{\uparrow}\mathrm{TV})\backslash(\mathrm{S}{\uparrow}\mathrm{TV}))/\hat{}(\mathrm{S}{\uparrow}\mathrm{TV})
\end{array}
$$

Applications of DLC: Quantification

John gave every book to Mary.

$$\forall x[(book\ x) \rightarrow (give\ m\ x\ j)]$$

$$\begin{array}{lll} \textbf{every} & - & \lambda x \lambda y \forall z[(x\ z) \rightarrow (y\ z)] \\ & := & ((\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S})/\mathrm{CN} \end{array}$$

- Eerst een noun: *book*
- Dan een zin met een NP gap: *John+gave+1+to+Mary*

Applications of DLC: Quantification

$$
\dfrac{\mathrm{CN} \Rightarrow \mathrm{CN} \qquad \dfrac{\dfrac{\mathrm{N}, \mathrm{PTV}, \mathrm{N}, \mathrm{PP} \Rightarrow \mathrm{S}}{\mathrm{N}, \mathrm{PTV}, [\,], \mathrm{PP} \Rightarrow \mathrm{S}{\uparrow}\mathrm{N}}\,{\uparrow}R \qquad \mathrm{S} \Rightarrow \mathrm{S}}{\mathrm{N}, \mathrm{PTV}, (\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S}, \mathrm{PP} \Rightarrow \mathrm{S}}\,{\downarrow}L}{\mathrm{N}, \mathrm{PTV}, ((\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S})/\mathrm{CN}, \mathrm{CN}, \mathrm{PP} \Rightarrow \mathrm{S}}\,/L
$$

Applications of DLC: Quantification

Mary thinks someone left.

$$
\dfrac{\dfrac{\dfrac{\mathrm{N}, \mathrm{N}\backslash\mathrm{S} \Rightarrow \mathrm{S}}{[\,], \mathrm{N}\backslash\mathrm{S} \Rightarrow \mathrm{S}{\uparrow}\mathrm{N}}\uparrow R \qquad \mathrm{S} \Rightarrow \mathrm{S}}{(\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S}, \mathrm{N}\backslash\mathrm{S} \Rightarrow \mathrm{S}}\downarrow L \qquad \mathrm{N}, \mathrm{N}\backslash\mathrm{S} \Rightarrow \mathrm{S}}{\mathrm{N}, (\mathrm{N}\backslash\mathrm{S})/\mathrm{S}, (\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S}, \mathrm{N}\backslash\mathrm{S} \Rightarrow \mathrm{S}}/L
$$

Applications of DLC: Quantification

Mary thinks someone left.

$$\dfrac{\dfrac{\mathrm{N}, (\mathrm{N}\backslash\mathrm{S})/\mathrm{S}, \mathrm{N}, \mathrm{N}\backslash\mathrm{S} \Rightarrow \mathrm{S}}{\mathrm{N}, (\mathrm{N}\backslash\mathrm{S})/\mathrm{S}, [\,], \mathrm{N}\backslash\mathrm{S} \Rightarrow \mathrm{S}{\uparrow}\mathrm{N}}\uparrow R \qquad \mathrm{S} \Rightarrow \mathrm{S}}{\mathrm{N}, (\mathrm{N}\backslash\mathrm{S})/\mathrm{S}, (\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S}, \mathrm{N}\backslash\mathrm{S} \Rightarrow \mathrm{S}}\downarrow L$$

Applications of DLC: Quantification

Everyone loves someone. $\quad \forall\exists$

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{N, (N\backslash S)/N, N \Rightarrow S}{N, (N\backslash S)/N, [\,] \Rightarrow S{\uparrow}N}\,{\uparrow}R
      \qquad S \Rightarrow S
    }{N, (N\backslash S)/N, (S{\uparrow}N){\downarrow}S \Rightarrow S}\,{\downarrow}L
  }{[\,], (N\backslash S)/N, (S{\uparrow}N){\downarrow}S \Rightarrow S{\uparrow}N}\,{\uparrow}R
  \qquad S \Rightarrow S
}{(S{\uparrow}N){\downarrow}S, (N\backslash S)/N, (S{\uparrow}N){\downarrow}S \Rightarrow S}\,{\downarrow}L
$$

Applications of DLC: Quantification

Everyone loves someone. $\exists\forall$

$$\cfrac{\cfrac{\cfrac{\cfrac{\mathrm{N}, (\mathrm{N}\backslash \mathrm{S})/\mathrm{N}, \mathrm{N} \Rightarrow \mathrm{S}}{[\,], (\mathrm{N}\backslash \mathrm{S})/\mathrm{N}, \mathrm{N} \Rightarrow \mathrm{S}{\uparrow}\mathrm{N}}\uparrow R \qquad \mathrm{S} \Rightarrow \mathrm{S}}{(\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S}, (\mathrm{N}\backslash \mathrm{S})/\mathrm{N}, \mathrm{N} \Rightarrow \mathrm{S}}\downarrow L}{(\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S}, (\mathrm{N}\backslash \mathrm{S})/\mathrm{N}, [\,] \Rightarrow \mathrm{S}{\uparrow}\mathrm{N}}\uparrow R \qquad \mathrm{S} \Rightarrow \mathrm{S}}{(\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S}, (\mathrm{N}\backslash \mathrm{S})/\mathrm{N}, (\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S} \Rightarrow \mathrm{S}}\downarrow L$$

Applications of NDLC: Complement alternation

John talked to Mary about Bill.

John talked about Bill to Mary.

$$\textbf{talked}{+}1{+}1 \quad - \quad \textit{talk}$$

$$:= \quad ((\mathrm{N}\backslash\mathrm{S})\uparrow\mathrm{PP}_{\mathrm{to}})\uparrow\mathrm{PP}_{\mathrm{about}}$$

Applications of NDLC: Complement alternation

$$\dfrac{\mathrm{PP}_{\mathrm{about}} \Rightarrow \mathrm{PP}_{\mathrm{about}} \qquad \dfrac{\mathrm{PP}_{\mathrm{to}} \Rightarrow \mathrm{PP}_{\mathrm{to}} \qquad \mathrm{VP} \Rightarrow \mathrm{VP}}{\sqrt[1]{VP{\uparrow}PP_{\mathrm{to}}}, \mathrm{PP}_{\mathrm{to}}, \sqrt[2]{VP{\uparrow}PP_{\mathrm{to}}} \Rightarrow \mathrm{VP}}\uparrow L}{\sqrt[1]{(VP{\uparrow}PP_{\mathrm{to}}){\uparrow}PP_{\mathrm{about}}}, \mathrm{PP}_{\mathrm{about}}, \sqrt[2]{(VP{\uparrow}PP_{\mathrm{to}}){\uparrow}PP_{\mathrm{about}}}, \mathrm{PP}_{\mathrm{to}}, \sqrt[3]{(VP{\uparrow}PP_{\mathrm{to}}){\uparrow}PP_{\mathrm{about}}} \Rightarrow \mathrm{VP}}\uparrow L$$

$$\dfrac{\mathrm{PP}_{\mathrm{about}} \Rightarrow \mathrm{PP}_{\mathrm{about}} tb \qquad \dfrac{\mathrm{PP}_{\mathrm{to}} \Rightarrow \mathrm{PP}_{\mathrm{to}} \qquad \mathrm{VP} \Rightarrow \mathrm{VP}}{\sqrt[1]{VP{\uparrow}PP_{\mathrm{to}}}, \mathrm{PP}_{\mathrm{to}}, \sqrt[2]{VP{\uparrow}PP_{\mathrm{to}}} \Rightarrow \mathrm{VP}}\uparrow L}{\sqrt[1]{(VP{\uparrow}PP_{\mathrm{to}}){\uparrow}PP_{\mathrm{about}}}, \mathrm{PP}_{\mathrm{to}}, \sqrt[2]{(VP{\uparrow}PP_{\mathrm{to}}){\uparrow}PP_{\mathrm{about}}}, \mathrm{PP}_{\mathrm{about}}, \sqrt[3]{(VP{\uparrow}PP_{\mathrm{to}}){\uparrow}PP_{\mathrm{about}}} \Rightarrow \mathrm{VP}}\uparrow L$$

Applications of NDLC: Particle shift

a. John called up Mary.

b. John called Mary up.

$$\begin{aligned} \textbf{called}+1+\textbf{up}+1 \quad &- \quad \textit{phone} \\ &:= \quad \check{}(N\backslash S)\uparrow N \end{aligned}$$

Applications of NDLC: Particle shift

$$\dfrac{\dfrac{\dfrac{\text{called up}}{\mathbf{called}+1+\mathbf{up}+1-\mathit{phone}{:}\,\check{}\,(\mathrm{N}\backslash \mathrm{S})\uparrow \mathrm{N}} \qquad \dfrac{\text{Mary}}{\mathbf{Mary}-m{:}\mathrm{N}}}{\mathbf{called}+1+\mathbf{up}+\mathbf{Mary}-(\mathit{phone}\ m){:}\,\check{}\,(\mathrm{N}\backslash \mathrm{S})}\,E\uparrow}{\mathbf{called}+\mathbf{up}+\mathbf{Mary}-(\mathit{phone}\ m){:}\mathrm{N}\backslash \mathrm{S}}\,E\check{}$$

$$\dfrac{\dfrac{\dfrac{\text{called up}}{\mathbf{called}+1+\mathbf{up}+1-\mathit{phone}{:}\,\check{}\,(\mathrm{N}\backslash \mathrm{S})\uparrow \mathrm{N}} \qquad \dfrac{\text{Mary}}{\mathbf{Mary}-m{:}\mathrm{N}}}{\mathbf{called}+\mathbf{Mary}+\mathbf{up}+1-(\mathit{phone}\ m){:}\,\check{}\,(\mathrm{N}\backslash \mathrm{S})}\,E\uparrow}{\mathbf{called}+\mathbf{Mary}+\mathbf{up}-(\mathit{phone}\ m){:}\mathrm{N}\backslash \mathrm{S}}\,E\check{}$$

Applications of NDLC: Cross-serial dependencies

a. ... dat Jan boeken las
that Jan books reads
"that Jan reads books"

b. ... dat Jan boeken kan lezen
that Jan books is-able read
"that Jan can read books"

c. ... dat Jan boeken wil kunnen lezen
that Jan books wants be-able read
"that Jan wants to be able to read books"

Applications of NDLC: Cross-serial dependencies

kan	–	*be-able*
	:=	$(N\backslash Sinf){\downarrow}(N\backslash S)$
1+**kunnen**	–	*be-able*
	:=	$(N\backslash Sinf){\downarrow}(N\backslash Sinf)$
las	–	*read*
	:=	$N\backslash(N\backslash S)$
1+**lezen**	–	*read*
	:=	$N\backslash(N\backslash Sinf)$
wil	–	*want*
	:=	$(N\backslash Sinf){\downarrow}(N\backslash S)$

Applications of NDLC: Cross-serial dependencies

a. ... dat Jan alles las
that Jan everything reads
"that Jan reads everything"

b. ... dat Jan alles kan lezen
that Jan everything is-able read
"that Jan can read everything"

Applications of NDLC: Cross-serial dependencies

$$
\begin{array}{rcl}
\textbf{kan} & - & \textit{be-able} \\
 & := & (N\backslash Sinf)\downarrow'(N\backslash S) \\
1'+\textbf{kunnen} & - & \textit{be-able} \\
 & := & (N\backslash Sinf)\downarrow'(N\backslash Sinf) \\
\textbf{las} & - & \textit{read} \\
 & := & N\backslash(N\backslash S) \\
1'+\textbf{lezen} & - & \textit{read} \\
 & := & N\backslash(N\backslash Sinf) \\
\textbf{wil} & - & \textit{want} \\
 & := & (N\backslash Sinf)\downarrow'(N\backslash S)
\end{array}
$$

Applications of NDLC: Cross-serial dependencies

$$\dfrac{\text{boeken}}{\textbf{boeken}-\textit{books}{:}\mathrm{N}} \qquad \dfrac{\text{lezen}}{1'+\textbf{lezen}-\textit{read}{:}\mathrm{N}\backslash(\mathrm{N}\backslash\mathrm{Sinf})}$$

$$\dfrac{\textbf{boeken}-\textit{books}{:}\mathrm{N} \quad 1'+\textbf{lezen}-\textit{read}{:}\mathrm{N}\backslash(\mathrm{N}\backslash\mathrm{Sinf})}{\textbf{boeken}+1'+\textbf{lezen}-(\textit{read books}){:}\mathrm{N}\backslash\mathrm{Sinf}}\,E\backslash \qquad \dfrac{\text{kunnen}}{1'+\textbf{kunnen}-\textit{be-able}{:}(\mathrm{N}\backslash\mathrm{Sinf})\downarrow'(\mathrm{N}\backslash\mathrm{Sinf})}$$

$$\dfrac{\textbf{boeken}+1'+\textbf{lezen}-(\textit{read books}){:}\mathrm{N}\backslash\mathrm{Sinf} \quad 1'+\textbf{kunnen}-\textit{be-able}{:}(\mathrm{N}\backslash\mathrm{Sinf})\downarrow'(\mathrm{N}\backslash\mathrm{Sinf})}{\textbf{boeken}+1'+\textbf{kunnen}+\textbf{lezen}-(\textit{be-able}\ (\textit{read books})){:}\mathrm{N}\backslash\mathrm{Sinf}}\,E\!\downarrow' \qquad \dfrac{\text{wil}}{\textbf{wil}-\textit{want}{:}(\mathrm{N}\backslash\mathrm{Sinf})\downarrow'(\mathrm{N}\backslash\mathrm{S})}$$

$$\dfrac{\textbf{boeken}+1'+\textbf{kunnen}+\textbf{lezen}-(\textit{be-able}\ (\textit{read books})){:}\mathrm{N}\backslash\mathrm{Sinf} \quad \textbf{wil}-\textit{want}{:}(\mathrm{N}\backslash\mathrm{Sinf})\downarrow'(\mathrm{N}\backslash\mathrm{S})}{\textbf{boeken}+\textbf{wil}+\textbf{kunnen}+\textbf{lezen}-(\textit{want}\ (\textit{be-able}\ (\textit{read books}))){:}\mathrm{N}\backslash\mathrm{S}}\,E\!\downarrow'$$

Vragen?

Einde